The original 19th century Waltz
This was a clockwise rotation ("natural turn"), later called the Rotary Waltz. This is the waltz of both Johann Strauss I and Johann Strauss Jr.'s eras.

The gent faces out of the hall, facing away from the center, in preparation. He turns a quarter CW (clockwise) before taking his first step, in order to build a little rotational momentum.

1: He steps L around his partner and ends up giving weight L by backing onto it. She turns to her right and steps R straight forward, slightly between his feet, toward LOD (Line Of Direction). At the end of ct 1, he is backing up, like a shield, in front of her,

2: He pulls his R foot back out of her way and gives weight onto it, tightly crossed behind his L. She takes a small step side L.

3: He steps L almost closed to R, continuing to rotate her. Thus on counts 1-2-3, his L foot remains almost on the same spot, pivoting. She steps R almost closed to L (or possibly crosses R slightly over in front of the L), continuing to rotate.

4-5-6: He does exactly what she did, and vice-versa.

19th century etiquette books for ladies advised that women actively rotate him as much as he's helping her turn. You can also help your partner rotate with a body lead, "looking to the right" with your upper body.

Elongated rotary waltz step
This was reportedly invented in Boston around 1875. It's similar to the Rotary Waltz except both you and your partner traveled somewhat toward LOD, with a side step, on each count 2 out of 3. Whomever is passing on the inside land takes the longer side step. Therefore she takes the longer side step on count 2, and he takes the longer side step on the next count 2 (i.e. count 5).

This is actually a rotating Box Step, which then spread to Europe, where it's done this way today. When it was first described in the early 1880s, it was called the New Waltz or the New Plain Waltz.

1880s Reverse Waltz
This is the CCW companion to the New Plain Waltz. The gent begins facing forward LOD.
1) He steps L forward to LOD, starting to help her rotate CCW, as she steps back R diagonally against LOD, a bit toward the center of the room. She shows her back to the center of the room.
2) She faces out and takes a small side step L toward LOD as he takes a long side step R toward LOD.
3) She closes her R to L, as he crosses his L very tightly over his R, taking weight on his L.

4-5-6) She does what he did, and vice versa.